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ε'/ε and Chiral Dynamics

ANTONIO PICH*

*Departament de Física Teòrica, IFIC, Universitat de Valencia – CSIC
 Edifici d'Instituts de Paterna, Apt. Correus 22085, E-46701 València, Spain*

The long-distance contributions to $K \rightarrow 2\pi$ amplitudes can be pinned down, using well established Chiral Perturbation Theory techniques. The strong S-wave rescattering of the two final pions generates sizeable chiral loop corrections, which have an important impact on the direct CP violation ratio ε'/ε [1,2]. Including all large logarithmic corrections, both at short and long distances, the Standard Model Prediction for this observable is found to be [2] $\text{Re}(\varepsilon'/\varepsilon) = (1.7 \pm 0.9) \cdot 10^{-3}$, in good agreement with the most recent experimental measurements. A better estimate of the strange quark mass could reduce the theoretical uncertainty to 30%.

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1 Introduction

The CP-violating ratio ε'/ε constitutes a fundamental test for our understanding of flavour-changing phenomena within the Standard Model framework. The experimental status has been clarified by the recent KTeV [3], $\text{Re}(\varepsilon'/\varepsilon) = (28.0 \pm 4.1) \cdot 10^{-4}$, and NA48 [4], $\text{Re}(\varepsilon'/\varepsilon) = (15.3 \pm 2.6) \cdot 10^{-4}$, measurements, which provide clear evidence for a non-zero value and, therefore, the existence of direct CP violation. The present world average is [3,4,5,6],

$$\text{Re}(\varepsilon'/\varepsilon) = (18.0 \pm 2.0) \cdot 10^{-4}, \quad (\chi^2/\text{ndf} = 10.8/3). \quad (1)$$

The theoretical prediction has been rather controversial since different groups, using different models or approximations, have obtained different results [7,8,9,10,11,12]. Although there was no universal agreement on the ε'/ε value predicted by the Standard Model, it has been often claimed that it is too small, failing to reproduce the experimental world average by at least a factor of two. This claim has generated a very intense theoretical activity, searching for new sources of CP violation beyond the Standard Model [13].

It has been pointed out recently [1] that the theoretical short-distance evaluations of ε'/ε had overlooked the important role of final-state interactions (FSI) in $K \rightarrow \pi\pi$ decays. Although it has been known for more than a decade that the rescattering of the two final pions induces a large correction to the isospin-zero decay amplitude, this effect was not taken properly into account in the theoretical predictions. From the measured π - π phase shifts one can easily infer [1] that FSI generate a strong enhancement of the ε'/ε prediction, by roughly the needed factor of two. This large correction is associated with infrared chiral logarithms involving the pion mass, which can be rigorously analyzed with standard Chiral Perturbation Theory (χ PT) techniques [14,15,16]. A very detailed analysis, including all large logarithmic corrections both at short and long distances, has been presented in ref. [2]. The resulting Standard Model prediction [2],

$$\text{Re}(\varepsilon'/\varepsilon) = (17 \pm 9) \cdot 10^{-4}, \quad (2)$$

is in good agreement with the most recent measurements.

The following sections present a brief overview of the most important ingredients entering the theoretical prediction of ε'/ε :

1. A short-distance calculation at the electroweak scale and its renormalization-group evolution to the three-flavour theory ($\mu \lesssim m_c$), which sums the leading and next-to-leading ultraviolet logarithms.
2. The matching to the χ PT description, which so far has been done at leading order in the $1/N_C$ expansion.
3. Chiral loop corrections, which induce large infrared logarithms related to FSI.

2 Theoretical framework

In terms of the $K \rightarrow \pi\pi$ isospin amplitudes, $\mathcal{A}_I = A_I e^{i\delta_I}$ ($I = 0, 2$),

$$\frac{\varepsilon'}{\varepsilon} = e^{i\Phi} \frac{\omega}{\sqrt{2}|\varepsilon|} \left[\frac{\text{Im}(A_2)}{\text{Re}(A_2)} - \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right]. \quad (3)$$

Owing to the well-known “ $\Delta I = 1/2$ rule”, ε'/ε is suppressed by the ratio $\omega = \text{Re}(A_2)/\text{Re}(A_0) \approx 1/22$. The strong S-wave rescattering of the two final pions generates a large phase-shift difference between the two isospin amplitudes, making the phases of ε' and ε nearly equal. Thus,

$$\Phi \approx \delta_2 - \delta_0 + \frac{\pi}{4} \approx 0. \quad (4)$$

The CP-conserving amplitudes $\text{Re}(A_I)$, their ratio ω and ε are usually set to their experimentally determined values. A theoretical calculation is then only needed for the quantities $\text{Im}(A_I)$.

One starts above the electroweak scale where the flavour-changing process, in terms of quarks, leptons and gauge bosons, can be analyzed within the usual gauge-coupling perturbative expansion in a rather straightforward way. Since M_Z is much larger than the long-distance hadronic scale M_K , there are large short-distance logarithmic contributions which can be summed up using the Operator Product Expansion (OPE) [17] and the renormalization group. The proper way to proceed makes use of modern Effective Field Theory (EFT) techniques [18].

The renormalization group is used to evolve down in energy from the electroweak scale, where the top quark and the Z and W^\pm bosons are integrated out. That means that one changes to a different EFT where those heavy particles are no longer explicit degrees of freedom. The new Lagrangian contains a tower of operators constructed with the light fields only, which scale as powers of $1/M_Z$. The information on the heavy fields is hidden in their (Wilson) coefficients, which are fixed by “matching” the high- and low-energy theories at the point $\mu = M_Z$. One follows the evolution further to lower energies, using the EFT renormalization group equations, until a new particle threshold is encountered. Then, the whole procedure of integrating the new heavy scale and matching to another EFT starts again.

One proceeds down to scales $\mu < m_c$ and gets finally an effective $\Delta S = 1$ Lagrangian, defined in the three-flavour theory [19,20],

$$\mathcal{L}_{eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} C_i(\mu) Q_i(\mu), \quad (5)$$

which is a sum of local four-fermion operators Q_i , constructed with the light degrees of freedom, modulated by Wilson coefficients $C_i(\mu)$ which are functions of the heavy

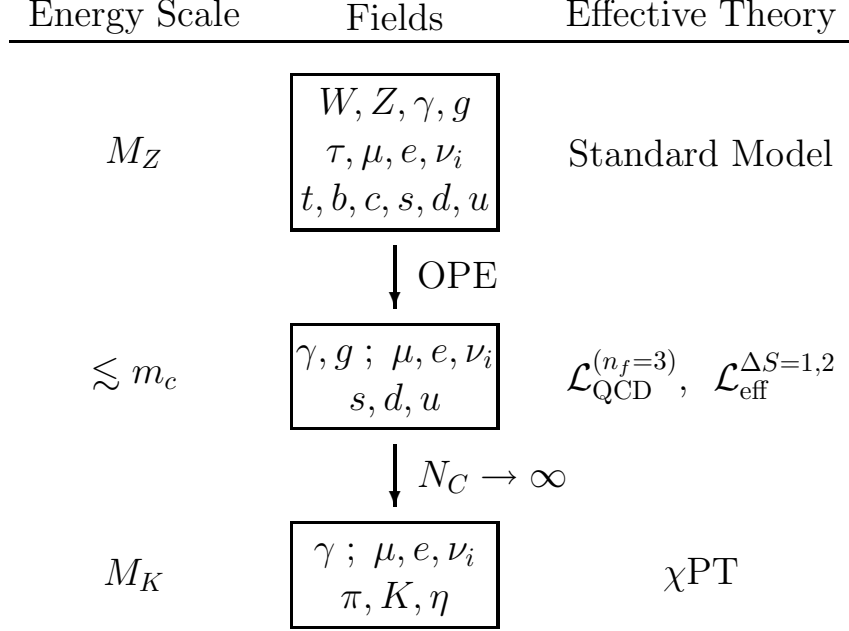


Figure 1: Evolution from M_Z to M_K .

masses. We have explicitly factored out the Fermi coupling G_F and the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements V_{ij} containing the usual Cabibbo suppression of K decays. The unitarity of the CKM matrix allows to write

$$C_i(\mu) = z_i(\mu) + \tau y_i(\mu), \quad (6)$$

where $\tau = -V_{td}V_{ts}^*/V_{ud}V_{us}^*$. Only the y_i components are needed to determine the CP-violating decay amplitudes. The overall renormalization scale μ separates the short- ($M > \mu$) and long- ($m < \mu$) distance contributions, which are contained in $C_i(\mu)$ and Q_i , respectively. The physical amplitudes are of course independent of μ .

Our knowledge of $\Delta S = 1$ transitions has improved qualitatively in recent years, thanks to the completion of the next-to-leading logarithmic order calculation of the Wilson coefficients [21,22]. All gluonic corrections of $O(\alpha_s^n t^n)$ and $O(\alpha_s^{n+1} t^n)$ are known, where $t \equiv \ln(M_1/M_2)$ refers to the logarithm of any ratio of heavy mass scales $M_1, M_2 \geq \mu$. Moreover, the full m_t/M_W dependence (at lowest order in α_s) is taken into account.

In order to predict physical amplitudes, however, one is still confronted with the calculation of hadronic matrix elements of the four-quark operators. This is a very difficult problem, which so far remains unsolved. Those matrix elements are usually parameterized in terms of the so-called bag parameters B_i , which measure them in units of their vacuum insertion approximation values.

To a very good approximation, the Standard Model prediction for ε'/ε can be written (up to global factors) as [7]

$$\frac{\varepsilon'}{\varepsilon} \sim \left[B_6^{(1/2)} (1 - \Omega_{IB}) - 0.4 B_8^{(3/2)} \right]. \quad (7)$$

Thus, only two operators are numerically relevant: the QCD penguin operator Q_6 ($\Delta I = 1/2$) governs $\text{Im}(A_0)$, while $\text{Im}(A_2)$ ($\Delta I = 3/2$) is dominated by the electroweak penguin operator Q_8 . The parameter

$$\Omega_{IB} = \frac{1}{\omega} \frac{\text{Im}(A_2)_{IB}}{\text{Im}(A_0)} \quad (8)$$

takes into account isospin-breaking corrections, which get enhanced by the large factor $1/\omega$.

The isospin-breaking correction coming from π^0 - η mixing was originally estimated to be $\Omega_{IB}^{\pi^0\eta} = 0.25$ [23,24]. Together with the usual ansatz $B_i \sim 1$, this produces a large numerical cancellation in eq. (7) leading to low values of ε'/ε around $7 \cdot 10^{-4}$. A recent improved calculation of π^0 - η mixing at $\mathcal{O}(p^4)$ in χ PT has found the result [25]

$$\Omega_{IB}^{\pi^0\eta} = 0.16 \pm 0.03. \quad (9)$$

This smaller number, slightly increases the naive estimate of ε'/ε .

3 Chiral Perturbation Theory

Below the resonance region one can use global symmetry considerations to define another EFT in terms of the QCD Goldstone bosons (π , K , η). The χ PT formulation of the Standard Model [14,15,16] describes the pseudoscalar-octet dynamics, through a perturbative expansion in powers of momenta and quark masses over the chiral symmetry breaking scale ($\Lambda_\chi \sim 1$ GeV).

Chiral symmetry fixes the allowed χ PT operators. At lowest order in the chiral expansion, the most general effective bosonic Lagrangian with the same $SU(3)_L \otimes SU(3)_R$ transformation properties as the short-distance Lagrangian (5) contains three terms, transforming as $(8_L, 1_R)$, $(27_L, 1_R)$ and $(8_L, 8_R)$, respectively. Their corresponding chiral couplings are denoted by g_8 , g_{27} and g_{ew} .

The tree-level $K \rightarrow \pi\pi$ amplitudes generated by the lowest-order χ PT Lagrangian do not contain any strong phases:

$$\begin{aligned} \mathcal{A}_0 &= -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sqrt{2} f_\pi \left\{ \left(g_8 + \frac{1}{9} g_{27} \right) (M_K^2 - M_\pi^2) - \frac{2}{3} f_\pi^2 e^2 g_8 g_{ew} \right\}, \\ \mathcal{A}_2 &= -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \frac{2}{9} f_\pi \left\{ 5 g_{27} (M_K^2 - M_\pi^2) - 3 f_\pi^2 e^2 g_8 g_{ew} \right\}. \end{aligned} \quad (10)$$

Taking the measured phase shifts into account, the moduli of g_8 and g_{27} can be extracted from the CP-conserving $K \rightarrow 2\pi$ decay rates; one gets [26] $|g_8| \approx 5.1$ and $|g_{27}| \approx 0.29$. The huge difference between these two couplings shows the well-known enhancement of octet $|\Delta I| = 1/2$ transitions. The g_{ew} term is the low-energy realization of the electroweak penguin operator.

The isospin amplitudes \mathcal{A}_I have been computed up to next-to-leading order in the chiral expansion [2,27,28,29,30,31]. The only remaining problem is the calculation of the χ PT couplings from the effective short-distance Lagrangian (5), which requires to perform the matching between the two EFTs. This can be easily done in the large- N_C limit of QCD [32], because in this limit the four-quark operators factorize into currents which have well-known chiral realizations. The local $\mathcal{O}(p^4)$ contributions to the amplitudes \mathcal{A}_I can be easily included in eqs. (10), through effective correction factors $\Delta_C \mathcal{A}_I^{(R)}$ to the lowest-order g_R contributions. At leading order in $1/N_C$, one gets [2]:

$$\begin{aligned}
g_8^\infty \left[1 + \Delta_C \mathcal{A}_0^{(8)}\right]^\infty &= \left\{ -\frac{2}{5} C_1(\mu) + \frac{3}{5} C_2(\mu) + C_4(\mu) - 16 L_5 C_6(\mu) \left[\frac{M_K^2}{(m_s + m_q)(\mu) f_\pi} \right]^2 \right\} f_0^{K\pi}(M_\pi^2), \\
g_{27}^\infty \left[1 + \Delta_C \mathcal{A}_0^{(27)}\right]^\infty &= g_{27}^\infty \left[1 + \Delta_C \mathcal{A}_2^{(27)}\right]^\infty = \frac{3}{5} [C_1(\mu) + C_2(\mu)] f_0^{K\pi}(M_\pi^2), \\
e^2 g_8^\infty \left[g_{ew} + \Delta_C \mathcal{A}_0^{(ew)}\right]^\infty &= -3 C_8(\mu) \left[\frac{M_K^2}{(m_s + m_q)(\mu) f_\pi} \right]^2 \left[1 + \frac{4L_5}{f_\pi^2} M_K^2 \right] \\
&\quad - \frac{3}{4} [C_7 - C_9 + C_{10}](\mu) \frac{M_K^2 - M_\pi^2}{f_\pi^2} f_0^{K\pi}(M_\pi^2), \\
e^2 g_8^\infty \left[g_{ew} + \Delta_C \mathcal{A}_2^{(ew)}\right]^\infty &= -3 C_8(\mu) \left[\frac{M_K^2}{(m_s + m_q)(\mu) f_\pi} \right]^2 \left[1 + \frac{4L_5}{f_\pi^2} M_\pi^2 \right] \\
&\quad + \frac{3}{2} [C_7 - C_9 - C_{10}](\mu) \frac{M_K^2 - M_\pi^2}{f_\pi^2} f_0^{K\pi}(M_\pi^2), \tag{11}
\end{aligned}$$

where $f_0^{K\pi}(M_\pi^2) \approx 1 + 4L_5 M_\pi^2/f_\pi^2$ is the $K\pi$ scalar form factor at the pion mass scale, L_5 is a coupling of the $\mathcal{O}(p^4)$ strong chiral Lagrangian and $m_q \equiv m_u = m_d$. In the limit $N_C \rightarrow \infty$, $L_5^\infty = \frac{1}{4} f_\pi^2 \left(\frac{f_K}{f_\pi} - 1 \right) / (M_K^2 - M_\pi^2) \approx 2.1 \cdot 10^{-3}$ and $f_0^{K\pi}(M_\pi^2) \approx 1.02$.

These results are equivalent to the usual large- N_C evaluations of the B_i factors. In particular, for ε'/ε where only the imaginary part of the g_R couplings matter [i.e. $\text{Im}(C_i)$] they amount to $B_8^{(3/2)} \approx B_6^{(1/2)} = 1$. Therefore, up to minor variations on some input parameters, the corresponding ε'/ε prediction reproduces the published results of the Munich [7] and Rome [8] groups.

The large- N_C limit is only applied to the matching between the 3-flavour quark theory and χ PT, as indicated in Figure 1. The evolution from the electroweak scale down to $\mu < m_c$ has to be done without any unnecessary expansion in powers of $1/N_C$; otherwise, one would miss large corrections of the form $\frac{1}{N_C} \ln(M/m)$, with $M \gg m$ two widely separated scales [33]. Thus, the Wilson coefficients contain the full μ dependence.

The large- N_C factorization of the four-quark operators Q_i ($i \neq 6, 8$) does not provide any scale dependence. Since the anomalous dimensions of these operators vanish when $N_C \rightarrow \infty$ [33], a very important ingredient is lost in this limit [34]. To achieve a reliable expansion in powers of $1/N_C$, one needs to go to the next order where this physics is captured [34,35]. This is the reason why the study of the $\Delta I = 1/2$ rule has proven to be so difficult. Fortunately, these operators are numerically suppressed in the ε'/ε prediction and their contributions can be easily corrected with the information provided by the measured CP-conserving rates [2,7].

The only anomalous dimensions which survive when $N_C \rightarrow \infty$ are precisely the ones corresponding to Q_6 and Q_8 [24,33]. One can then expect that the matrix elements of these two operators are well approximated by this limit [34,35,36]. These operators factorize into colour-singlet scalar and pseudoscalar currents, which are μ dependent. This generates the factors $\langle \bar{q}q \rangle(\mu) \approx -f_\pi^2 M_K^2 / (m_s + m_q)(\mu)$, which exactly cancel the μ dependence of $C_{6,8}(\mu)$ at large N_C [24,33,34,35,36,37]. It remains of course a dependence at next-to-leading order.

Therefore, while there are large $1/N_C$ corrections to $\text{Re}(g_I)$ [35], the large- N_C limit is expected to give a good estimate of $\text{Im}(g_I)$.

4 Chiral loop corrections

The lowest-order calculation does not provide any strong phases δ_I . Those phases originate in the final rescattering of the two pions and, therefore, are generated by chiral loops which are of higher order in both the momentum and $1/N_C$ expansions. Analyticity and unitarity require the presence of a corresponding dispersive FSI effect in the moduli of the isospin amplitudes. Since the S-wave strong phases are quite large, specially in the isospin-zero case, one should expect large higher-order unitarity corrections.

The size of the FSI effect can be calculated at one loop in χ PT. The dominant one-loop correction to the octet amplitude comes indeed from the elastic soft rescattering of the two pions in the final state. The existing one-loop analyses [2,27,28] show that pion loop diagrams provide an important enhancement of the \mathcal{A}_0 amplitude, implying a corresponding reduction of the phenomenologically fitted value of $|g_8|$. This chiral loop correction destroys the accidental numerical cancellation in eq. (7), generating a sizeable enhancement of the ε'/ε prediction [1].

Let us decompose the isospin amplitudes in their different chiral components as $\mathcal{A}_0 = \mathcal{A}_0^{(8)} + \mathcal{A}_0^{(27)} + \mathcal{A}_0^{(ew)}$ and $\mathcal{A}_2 = \mathcal{A}_2^{(27)} + \mathcal{A}_2^{(ew)}$. Moreover, we can write them in the form

$$\mathcal{A}_I^{(R)} = \mathcal{A}_I^{(R)\infty} \times \mathcal{C}_I^{(R)}, \quad (12)$$

where $\mathcal{A}_I^{(R)\infty}$ are the large- N_C results obtained in the previous section. The correction factors $\mathcal{C}_I^{(R)} \equiv 1 + \Delta_L \mathcal{A}_I^{(R)}$ contain the chiral loop contributions $\Delta_L \mathcal{A}_I^{(R)}$ that we are interested in. Their complete analytical expressions at one loop in χ PT have been given in ref. [2], where the following numerical values have been obtained:

$$\begin{aligned} \mathcal{C}_0^{(8)} &= 1.27 \pm 0.05 + 0.46 i, \\ \mathcal{C}_0^{(27)} &= 2.0 \pm 0.7 + 0.46 i, \\ \mathcal{C}_0^{(ew)} &= 1.27 \pm 0.05 + 0.46 i, \\ \mathcal{C}_2^{(27)} &= 0.96 \pm 0.05 - 0.20 i, \\ \mathcal{C}_2^{(ew)} &= 0.50 \pm 0.24 - 0.20 i. \end{aligned} \quad (13)$$

The central values have been evaluated at the chiral renormalization scale $\nu = M_\rho$. To estimate the corresponding uncertainties we have allowed the scale ν to change between 0.6 and 1 GeV. The scale dependence is only present in the dispersive contributions and should cancel with the corresponding ν dependence of the local χ PT counterterms. However, this dependence is next-to-leading in $1/N_C$ and, therefore, is not included in the large- N_C determination of the chiral couplings. The sensitivity of the results to the scale ν gives a good estimate of those missing contributions. Notice that all amplitudes with a given isospin get the same absorptive contribution, as it should since they have identical strong phase shifts.

The numerical corrections to the 27-plet amplitudes do not have much phenomenological interest for CP-violating observables, because $\text{Im}(g_{27}) = 0$. Remember that the CP-conserving amplitudes $\text{Re}(A_I)$ are set to their experimentally determined values. What is relevant for the ε'/ε prediction is the 35% enhancement of the isoscalar octet amplitude $\text{Im}[A_0^{(8)}]$ and the 46% reduction of $\text{Im}[A_2^{(ew)}]$. Just looking to the simplified formula (7), one realizes immediately the obvious impact of these one-loop chiral corrections.

A complete $\mathcal{O}(p^4)$ calculation [25,29] of the isospin-breaking parameter Ω_{IB} is not yet available. The value 0.16 quoted in eq. (9) only accounts for the contribution from π^0 - η mixing [25] and should be corrected by the effect of chiral loops. Since $|\mathcal{C}_2^{(27)}| \approx 0.98 \pm 0.05$, one does not expect any large correction of $\text{Im}(A_2)_{IB}$, while we know that $\text{Im}[A_0^{(8)}]$ gets enhanced by a factor 1.35. Taking this into account, one gets the corrected value

$$\Omega_{IB} \approx \Omega_{IB}^{\pi^0\eta} \left| \frac{\mathcal{C}_2^{(27)}}{\mathcal{C}_0^{(8)}} \right| = 0.12 \pm 0.05, \quad (14)$$

where the quoted error is an educated theoretical guess. This value agrees with the result $\Omega_{IB} = 0.08 \pm 0.05 \pm 0.01$, obtained in ref. [38] by using three different models [9,31,35,39,40,41] to estimate the relevant $\mathcal{O}(p^4)$ chiral couplings.

The sensitivity to higher-order chiral loop corrections has been investigated in ref. [2] through an Omnès exponentiation of the dominant pion loops [1], using the experimental $\pi\pi$ phase shifts. The standard one-loop χ PT results and the Omnès calculation agree within errors, indicating a good convergence of the chiral expansion.

5 Numerical results and discussion

The infrared effect of chiral loops generates an important enhancement of the isoscalar $K \rightarrow \pi\pi$ amplitude. This effect gets amplified in the prediction of ε'/ε , because at lowest order (in both $1/N_C$ and the chiral expansion) there is an accidental numerical cancellation between the $I = 0$ and $I = 2$ contributions. Since the chiral loop corrections destroy this cancellation, the final result for ε'/ε is dominated by the isoscalar amplitude. Thus, the Standard Model prediction for ε'/ε is finally governed by the matrix element of the gluonic penguin operator Q_6 .

A detailed numerical analysis has been provided in ref. [2]. The short-distance Wilson coefficients have been evaluated at the scale $\mu = 1$ GeV. Their associated uncertainties have been estimated through the sensitivity to changes of μ in the range $M_\rho < \mu < m_c$ and to the choice of γ_5 scheme. Since the most important α_s corrections appear at the low-energy scale μ , the strong coupling has been fixed at the τ mass, where it is known [42] with about a few percent level of accuracy: $\alpha_s(m_\tau) = 0.345 \pm 0.020$. The values of α_s at the other needed scales can be deduced through the standard renormalization group evolution.

Taking the experimental value of ε , the CP-violating ratio ε'/ε is proportional to the CKM factor $\text{Im}(V_{ts}^* V_{td}) = (1.2 \pm 0.2) \cdot 10^{-4}$ [43]. This number is sensitive to the input values of several non-perturbative hadronic parameters adopted in the usual unitarity triangle analysis; thus, it is subject to large theoretical uncertainties which are difficult to quantify [44]. Using instead the theoretical prediction of ε , this CKM factor drops out from the ratio ε'/ε ; the sensitivity to hadronic inputs is then reduced to the explicit remaining dependence on the $\Delta S = 2$ scale-invariant bag parameter \hat{B}_K . In the large- N_C limit, $\hat{B}_K = 3/4$. We have performed the two types of numerical analysis, obtaining consistent results. This allows us to estimate better the theoretical uncertainties, since the two analyses have different sensitivity to hadronic inputs.

The final result quoted in ref. [2] is:

$$\text{Re}(\varepsilon'/\varepsilon) = (1.7 \pm 0.2 \substack{+0.8 \\ -0.5} \pm 0.5) \cdot 10^{-3} = (1.7 \pm 0.9) \cdot 10^{-3}. \quad (15)$$

The first error comes from the short-distance evaluation of Wilson coefficients and the choice of low-energy matching scale μ . The uncertainty coming from varying the strange quark mass in the interval $(m_s + m_q)(1 \text{ GeV}) = 156 \pm 25 \text{ MeV}$

[45,46,47,48,49,50,51] is indicated by the second error. The most critical step is the matching between the short- and long-distance descriptions. We have performed this matching at leading order in the $1/N_C$ expansion, where the result is known to $\mathcal{O}(p^4)$ and $\mathcal{O}(e^2 p^2)$ in χ PT. This can be expected to provide a good approximation to the matrix elements of the leading Q_6 and Q_8 operators. Since all ultraviolet and infrared logarithms have been resummed, our educated guess for the theoretical uncertainty associated with $1/N_C$ corrections is $\sim 30\%$ (third error).

Thus, a better determination of the strange quark mass would allow to reduce the uncertainty to the 30% level. In order to get a more accurate prediction, it would be necessary to have a good analysis of next-to-leading $1/N_C$ corrections. This is a very difficult task, but progress in this direction can be expected in the next few years [9,11,35,52,53,54].

To summarize, using a well defined computational scheme, it has been possible to pin down the value of ε'/ε with an acceptable accuracy. Within the present uncertainties, the resulting Standard Model theoretical prediction (15) is in good agreement with the measured experimental value (1), without any need to invoke a new physics source of CP violation.

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